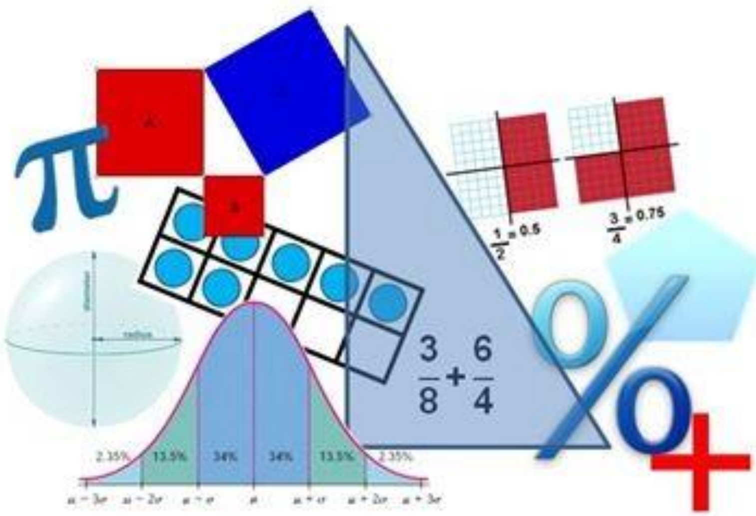
Mathematics

2016 Standards of Learning

Grade 4

Curriculum Framework



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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades three through five should continue to foster the development of number sense, with greater emphasis on decimals and fractions. Students with good number sense understand the meaning of numbers, develop multiple relationships and representations among numbers, and recognize the relative magnitude of numbers. They should learn the relative effect of operating on whole numbers, fractions, and decimals and learn how to use mathematical symbols and language to represent problem situations. Number and operation sense continues to be the cornerstone of the curriculum.

The focus of instruction in grades three through five allows students to investigate and develop an understanding of number sense by modeling numbers, using different representations (e.g., physical materials, diagrams, mathematical symbols, and word names), and making connections among mathematics concepts as well as to other content areas. Students should develop strategies for reading, writing, and judging the size of whole numbers, fractions, and decimals by comparing them, using a variety of models and benchmarks as referents (e.g., $\frac{1}{2}$ or 0.5). Students should apply their knowledge of number and number sense to investigate and solve a variety of problem types.

4.1 The student will
a) read, write, and identify the place and value of each digit in a nine-digit whole number;
b) compare and order whole numbers expressed through millions; and
c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
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- The structure of the base-ten number system is based upon a simple pattern of tens, in which the value of each place is ten times the value of the place to its right.
- Place value refers to the value of each digit and depends upon the position of the digit in the number. For example, in the number 7,864,352, the 8 is in the hundred thousand place, and the value of the 8 is eight hundred thousand or 800,000.
- Whole numbers may be written in a variety of forms:
  - Standard: 1,234,567
  - Written: one million, two hundred thirty-four thousand, five hundred sixty-seven
  - Expanded: (1,000,000 + 200,000 + 30,000 + 4,000 + 500 + 60 + 7)
- Numbers are arranged into groups of three places called *periods* (ones, thousands, millions). The value of the places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the value of the place and period of a number helps students determine values of digits in any number as well as read and write numbers. Students at this level will work with numbers through the millions period (nine-digit numbers).
- Reading and writing large numbers should be meaningful for students. Experiences can be provided that relate practical situations (e.g., numbers found in the students' environment including population, number of school lunches sold statewide in a day, etc.).
- Concrete materials such as base-ten blocks or bundles of sticks may be used to represent whole numbers through thousands. Larger numbers may be represented by digit cards and place value charts or on number lines.
- Number lines are useful tools when developing a conceptual understanding of rounding with whole numbers. When given a number to round, locate it on the number line. Next, determine the closest multiples of thousand, ten-thousand, or hundred-thousand it is between. Then, identify to which it is closer.
- Mathematical symbols (>, <) used to compare two unequal numbers are called *inequality symbols*.

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The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Read nine-digit whole numbers, presented in standard form and represent the same number in written form. (a)
- Write nine-digit whole numbers in standard form when the numbers are presented orally or in written form. (a)
- Identify and communicate, orally and in written form, the place and value for each digit in a nine-digit whole number. (a)
- Compare two whole numbers expressed through millions, using the words *greater than, less than, equal to,* and *not equal to* or using the symbols >, <, =, or ≠. (b)
- Order up to four whole numbers expressed through millions. (b)
- Round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand place. (c)
- Identify the range of numbers that round to a given thousand, ten thousand, and hundred thousand. (c)

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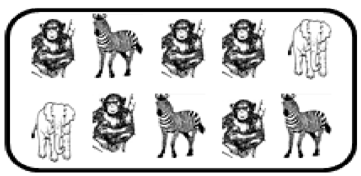
4.2 The student will

a) compare and order fractions and mixed numbers, with and without models;*

b) represent equivalent fractions;* and

c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A fraction is a way of representing part of a whole region (i.e., an area model), part of a group (i.e., a set model), or part of a length (i.e., a measurement model). • In the area and length/measurement fraction models, the parts must be equivalent. • In a set model, each member of the set is an equivalent part of the set. In set models, the whole needs to be defined, but members of the set may have different sizes and shapes. For instance, if a whole is defined as a set of 10 animals, the animals within the set may be different. For example, students should be able to identify monkeys as representing $\frac{1}{2}$ of the animals in the following set.  • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). • The value of a fraction is dependent on both the number of equivalent parts in a whole (denominator) and the number of those parts being considered (numerator). • The more parts the whole is divided into, the smaller the parts (e.g., $\frac{1}{5} < \frac{1}{3}$). • When fractions have the same denominator, they are said to have "common denominators" or "like denominators." Comparing fractions with like denominators involves comparing only the numerators.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and order no more than four fractions having like and unlike denominators of 12 or less, using concrete and pictorial models. (a) • Use benchmarks (e.g., 0, $\frac{1}{2}$ or 1) to compare and order no more than four fractions having unlike denominators of 12 or less. (a) • Compare and order no more than four fractions with like denominators of 12 or less by comparing number of parts (numerators) (e.g., $\frac{1}{5} < \frac{3}{5}$). (a) • Compare and order no more than four fractions with like numerators and unlike denominators of 12 or less by comparing the size of the parts (e.g., $\frac{3}{9} < \frac{3}{5}$). (a) • Compare and order no more than four fractions (proper or improper), and/or mixed numbers, having denominators of 12 or less. (a) • Use the symbols >, <, =, and ≠ to compare fractions (proper or improper) and/or mixed numbers having denominators of 12 or less. (a) • Represent equivalent fractions through twelfths, using region/area models, set models, and measurement/length models. (b)

4.2 The student will
a) compare and order fractions and mixed numbers, with and without models;*
b) represent equivalent fractions;* and
c) identify the division statement that represents a fraction, with models and in context.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Strategies for comparing fractions having unlike denominators may include: – comparing fractions to familiar benchmarks (e.g., 0, $\frac{1}{2}$, 1); – determining equivalent fractions, using models such as fraction strips, number lines, fraction circles, rods, pattern blocks, cubes, base-ten blocks, tangrams, graph paper, or patterns in a multiplication chart; and – determining a common denominator by determining the least common multiple (LCM) of both denominators and then rewriting each fraction as an equivalent fraction, using the LCM as the denominator. • A variety of fraction models should be used to expand students' understanding of fractions and mixed numbers: – Region/area models: a surface or area is subdivided into smaller equal parts, and each part is compared with the whole (e.g., fraction circles, pattern blocks, geoboards, grid paper, color tiles). – Set models: the whole is understood to be a set of objects, and subsets of the whole make up fractional parts (e.g., counters, chips). – Measurement models: similar to area models but lengths instead of areas are compared (e.g., fraction strips, rods, cubes, number lines, rulers). • Equivalent fractions name the same amount. Students should use a variety of representations and models to identify different names for equivalent fractions. • When presented with a fraction $\frac{3}{5}$ representing division, the division expression representing the fraction is written as 3 ÷ 5. • The fraction $\frac{3}{4}$ may be interpreted as the amount of cake each person will receive when 3 cakes are divided equally among 4 people.	• Identify the division statement that represents a fraction with models and in context (e.g., $\frac{3}{5}$ means the same as 3 divided by 5 or $\frac{3}{5}$ represents the amount of muffin each of five children will receive when sharing 3 muffins equally). (c)

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number;
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.*

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Decimal numbers expand the set of whole numbers and, like fractions, are a way of representing part of a whole. • The structure of the base-ten number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship (e.g., in 2.35, 3 is in the tenths place since it takes ten one-tenths to make one whole). Use base-ten proportional manipulatives, such as place value mats/charts, decimal squares, base-ten blocks, meter sticks, as well as the ten-to-one non-proportional model, money, to investigate this relationship. • A decimal point separates the whole number places from the places that are less than one. A number containing a decimal point is called a *decimal number* or simply a *decimal*. • To read decimals, – read the whole number to the left of the decimal point; – read the decimal point as "and"; – read the digits to the right of the decimal point just as you would read a whole number; and – say the name of the place value of the digit in the smallest place. • Any decimal less than 1 will include a leading zero. For example 0.125 which can be read as "zero and one hundred twenty-five thousandths" or as "one hundred twenty-five thousandths." • Decimals may be written in a variety of forms: – Standard: 26.537 – Written: twenty-six and five hundred thirty-seven thousandths – Expanded: 20 + 6 + 0.5 + 0.03 + 0.007. • Strategies for rounding whole numbers can be applied to rounding decimals.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read and write decimals expressed through thousandths, using base-ten manipulatives, drawings, and numerical symbols. (a) • Represent and identify decimals expressed through thousandths, using base-ten manipulatives, pictorial representations, and numerical symbols (e.g., relate the appropriate drawing to 0.05). (a) • Investigate the ten-to-one place value relationship for decimals through thousandths, using base-ten manipulatives (e.g., place value mats/charts, decimal squares, and base-ten blocks). (a) • Identify and communicate, both orally and in written form, the position and value of a decimal through thousandths (e.g., given 0.385, the 8 is in the hundredths place and has a value of 0.08. (a) • Round decimals expressed through thousandths to the nearest whole number. (b) • Compare two decimals expressed through thousandths, using symbols (>, <, =, and ≠) and/or words (*greater than, less than, equal to,* and *not equal to*). (c) • Order a set of up to four decimals, expressed through thousandths, from least to greatest or greatest to least. (c)

4.3 The student will

a) read, write, represent, and identify decimals expressed through thousandths;
b) round decimals to the nearest whole number;
c) compare and order decimals; and
d) given a model, write the decimal and fraction equivalents.*

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Understanding the Standard	Essential Knowledge and Skills
• Number lines are useful tools when developing a conceptual understanding of rounding with decimals. When given a decimal to round to the nearest whole or ones place, locate it on the number line. Next, determine the two whole numbers it is between. Then, identify to which it is closer. • Base-ten models concretely relate fractions to decimals (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money). • Decimals and fractions represent the same relationships; however, they are presented in two different forms. The decimal 0.25 is written as $\frac{1}{4}$. Decimal numbers are another way of writing fractions.	• Represent fractions for halves, fourths, fifths, and tenths as decimals through hundredths, using concrete objects. (d) • Relate fractions to decimals, using concrete objects (e.g., 10-by-10 grids, meter sticks, number lines, decimal squares, decimal circles, money). (d) • Write the decimal and fraction equivalent for a given model (e.g., $\frac{1}{4}$ = 0.25 or 0.25 = $\frac{1}{4}$; 1.25 = $\frac{5}{4}$ or $1\frac{1}{4}$). (d)

Computation and estimation in grades three through five should focus on developing fluency in multiplication and division with whole numbers and should begin to extend students' understanding of these operations to work with decimals. Instruction should focus on computation activities that enable students to model, explain, and develop proficiency with basic facts and algorithms. These proficiencies are often developed as a result of investigations and opportunities to develop algorithms. Additionally, opportunities to develop and use visual models, benchmarks, and equivalents, to add and subtract fractions, and to develop computational procedures for the addition and subtraction of decimals are a priority for instruction in these grades. Multiplication and division with decimals will be explored in grade five.

Students should develop an understanding of how whole numbers, fractions, and decimals are written and modeled; an understanding of the meaning of multiplication and division, including multiple representations (e.g., multiplication as repeated addition or as an array); an ability to identify and use relationships among operations to solve problems (e.g., multiplication as the inverse of division); and the ability to use properties of operations to solve problems (e.g., 7×28 is equivalent to $(7 \times 20) + (7 \times 8)$).

Students should develop computational estimation strategies based on an understanding of number concepts, properties, and relationships. Practice should include estimation of sums and differences of common fractions and decimals, using benchmarks (e.g., $\frac{2}{5} + \frac{1}{3}$ must be less than 1 because both fractions are less than $\frac{1}{2}$). Using estimation, students should develop strategies to recognize the reasonableness of their solutions.

Additionally, students should enhance their ability to select an appropriate problem-solving method from among estimation, mental mathematics, paper-and-pencil algorithms, and the use of calculators and computers. With activities that challenge students to use this knowledge and these skills to solve problems in many contexts, students develop the foundation to ensure success and achievement in higher mathematics.

4.4 The student will

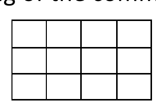
a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
b) estimate and determine sums, differences, and products of whole numbers;*
c) estimate and determine quotients of whole numbers, with and without remainders;* and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator**.**

Understanding the Standard

- Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently.
- The development of computational fluency relies on quick access to number facts. There are patterns and relationships that exist in the facts. These relationships can be used to learn and retain the facts.
- A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil.
- In grade three, students developed an understanding of the meanings of multiplication and division of whole numbers through activities and practical problems involving equal-sized groups, arrays, and length models. In addition, grade three students have worked on fluency of facts for 0, 1, 2, 5, and 10.
- Three models used to develop an understanding of multiplication include:
  – The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces the concept of multiplication as a way to find the total number of items in a collection of groups, with the same amount in each group, and the total number of items can be found by repeated addition or skip counting.

  – The array model, consisting of rows and columns (e.g., three rows of four columns for a 3-by-4 array), helps build an understanding of the commutative property.



Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Demonstrate fluency with multiplication through 12 × 12, and the corresponding division facts. (a)
- Estimate whole number sums, differences, products, and quotients, with and without context. (b, c)
- Apply strategies, including place value and the properties of addition to determine the sum or difference of two whole numbers, each 999,999 or less. (b)
- Apply strategies, including place value and the properties of multiplication and/or addition, to determine the product of two whole numbers when both factors have two digits or fewer. (b)
- Apply strategies, including place value and the properties of multiplication and/or addition, to determine the quotient of two whole numbers, given a one-digit divisor and a two- or three-digit dividend, with and without remainders. (c)
- Refine estimates by adjusting the final amount, using terms such as *closer to, between*, and *a little more than*. (b, c)
- Create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication with whole numbers. (d)

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a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
b) estimate and determine sums, differences, and products of whole numbers;*
c) estimate and determine quotients of whole numbers, with and without remainders;* and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

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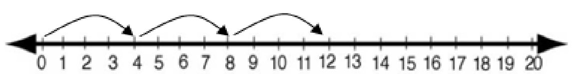
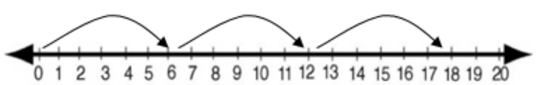
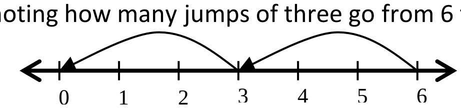
Understanding the Standard	Essential Knowledge and Skills
– The length model (e.g., a number line) also reinforces repeated addition or skip counting.  • There is an inverse relationship between multiplication and division. • The number line model can be used to solve a multiplication problem such as 3 × 6. This is represented on the number line by three jumps of six or six jumps of three, depending on the context of the problem.  • The number line model can be used to solve a division problem such as 6 ÷ 3 and is represented on the number line by noting how many jumps of three go from 6 to 0.  • The number line model above shows two jumps of three between 6 and 0, answering the question of how many jumps of three go from 6 to 0; therefore, 6 ÷ 3 = 2. • In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, a hundreds chart, and mental mathematics. Strategies to learn the multiplication facts include an understanding of multiples, properties of zero and one as factors, commutative property, and related facts. Investigating arithmetic operations with whole numbers helps students learn about the different properties of arithmetic relationships. These relationships remain true regardless of the whole numbers. • Grade four students should explore and apply the properties of addition and multiplication as strategies for solving addition, subtraction, multiplication, and division problems using a variety of representations (e.g., manipulatives, diagrams, and symbols).	• Create and solve single-step practical problems involving division with whole numbers. (d) • Use the context in which a practical problem is situated to interpret the quotient and remainder. (d)

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b) estimate and determine sums, differences, and products of whole numbers;*
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d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• The properties of the operations are "rules" about how numbers work and how they relate to one another. Students at this level do not need to use the formal terms for these properties but should utilize these properties to further develop flexibility and fluency in solving problems. The following properties are most appropriate for exploration at this level: – The identity property of addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. – The commutative property of addition states that changing the order of the addends does not affect the sum (e.g., 24 + 136 = 136 + 24). Similarly, the commutative property of multiplication states that changing the order of the factors does not affect the product (e.g., 12 × 43 = 43 × 12). – The associative property of addition states that the sum stays the same when the grouping of addends is changed (e.g., 15 + (35 + 16) = (15 + 35) + 16). The associative property of multiplication states that the product stays the same when the grouping of factors is changed [e.g., 16 × (40 × 5) = (16 × 40) × 5]. – The distributive property states that multiplying a sum by a number gives the same result as multiplying each addend by the number and then adding the products. Several examples are shown below: – 3(9) = 3(5 + 4) 3(5 + 4) = (3 × 5) + (3 × 4) – 5 × (3 + 7) = (5 × 3) + (5 × 7) – (2 × 3) + (2 × 5) = 2 × (3 + 5)	

4.4 **The student will**

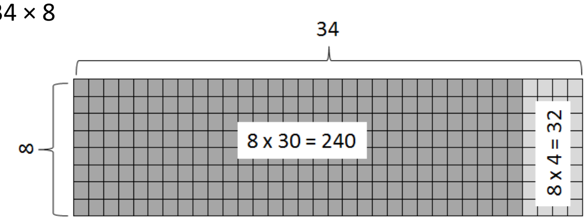
a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
b) estimate and determine sums, differences, and products of whole numbers;*
c) estimate and determine quotients of whole numbers, with and without remainders;* and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

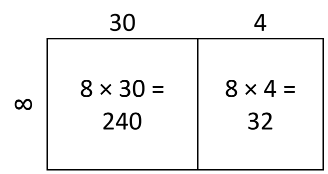
*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills

– 9 × 23
9(20+3)
180 + 27
207

– 34 × 8





- Addition is the combining of quantities; it uses the following terms:

addend	→	45,623
addend	→	+ 37,846
sum	→	83,469

- Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms:

minuend	→	45,698
subtrahend	→	– 32,741
difference	→	12,957

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*

b) estimate and determine sums, differences, and products of whole numbers;*

c) estimate and determine quotients of whole numbers, with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• The terms associated with multiplication are listed below: $\begin{array}{lcr} \textit{factor} & \rightarrow & 76 \\ \textit{factor} & \rightarrow & \underline{\times\ 23} \\ \textit{product} & \rightarrow & 1{,}748 \end{array}$ • In multiplication, one factor represents the number of equal groups and the other factor represents the number in or size of each group. The product is the total number in all of the groups. • Multiplication can also refer to a multiplicative comparison, such as: "Gwen has six times as many stickers as Phillip". Both situations should be modeled with manipulatives. • Models of multiplication may include repeated addition and collections of like sets, partial products, and area or array models. • Division is the operation of making equal groups or shares. When the original amount and the number of shares are known, divide to determine the size of each share. When the original amount and the size of each share are known, divide to determine the number of shares. Both situations may be modeled with base-ten manipulatives. • Division is the inverse of multiplication. Terms used in division are *dividend, divisor*, and *quotient.* $\textit{dividend} \div \textit{divisor} = \textit{quotient} \qquad \textit{divisor}\overline{\smash{)}\,\textit{dividend}}^{\;\textit{quotient}} \qquad \frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient}$ • Students benefit from experiences with various methods of division, such as repeated subtraction and partial quotients.	

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
b) estimate and determine sums, differences, and products of whole numbers;*
c) estimate and determine quotients of whole numbers, with and without remainders;* and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Estimation can be used to determine the approximation for and then to verify the reasonableness of sums, differences, products, and quotients of whole numbers. An estimate is a number that lies within a range of the exact solution, and the estimation strategy used in a particular problem determines how close the number is to the exact solution. An estimate tells about how much or about how many. • Strategies such as rounding up or down, front-end, and compatible numbers may be used to estimate sums, differences, products, and quotients of whole numbers. • The least number of steps necessary to solve a single-step problem is one. • The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc., encourages students to perform a particular operation rather than make sense of the context of the problem. A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses. • Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. • Students should experience a variety of problem types related to multiplication and division. Some examples are included in the following chart:	

4.4 **The student will**

a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*
b) estimate and determine sums, differences, and products of whole numbers;*
c) estimate and determine quotients of whole numbers, with and without remainders;* and
d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills

<table>
<tr><th colspan="6">GRADE 4: COMMON MULTIPLICATION AND DIVISION PROBLEM TYPES</th></tr>
<tr><td colspan="6">Equal Group Problems</td></tr>
<tr><td colspan="2">Whole Unknown (Multiplication)</td><td colspan="2">Size of Groups Unknown (Partitive Division)</td><td colspan="2">Number of Groups Unknown (Measurement Division)</td></tr>
<tr><td colspan="2">There are six boxes of crayons. Each box contains 24 crayons. How many crayons are there in all?</td><td colspan="2">If 144 crayons are shared equally among six friends, how many crayons will each friend get?</td><td colspan="2">If 144 crayons are placed into school boxes with each box containing 24 crayons, how many school boxes can be filled?</td></tr>
<tr><td colspan="6">Multiplicative Comparison Problems</td></tr>
<tr><td colspan="2">Result Unknown</td><td colspan="2">Start Unknown</td><td colspan="2">Comparison Factor Unknown</td></tr>
<tr><td colspan="2">Tyrone ran 30 miles last month. Jasmine ran four times as many miles as Tyrone during the same month. How many miles did Jasmine run?</td><td colspan="2">Jasmine ran 120 miles. She ran four times as many miles as Tyrone. How many miles did Tyrone run?</td><td colspan="2">Jasmine ran 120 miles. Tyrone ran 30 miles. How many times more miles did Jasmine run than Tyrone?</td></tr>
<tr><td colspan="6">Array or Area Problems</td></tr>
<tr><td colspan="3">Whole Unknown</td><td colspan="3">One Dimension Unknown</td></tr>
<tr><td colspan="3">There are 12 baseball teams competing in the tournament. Each team has 9 baseball players. How many baseball players are there all together?

Mr. Myers’s dog pen measures 15 feet by 22 feet. How many square feet are in the dog pen?</td><td colspan="3">There are 108 baseball players competing in the tournament. The players are divided equally among 12 teams. How many players are on each team?

There are 108 baseball players competing in the tournament. There are exactly 9 players on each team. How many teams are there?

The dog pen covers 60 square feet. The length of the dog pen is 15 feet. What is the width of the dog pen?</td></tr>
</table>

4.4 The student will

a) demonstrate fluency with multiplication facts through 12 × 12, and the corresponding division facts;*

b) estimate and determine sums, differences, and products of whole numbers;*

c) estimate and determine quotients of whole numbers, with and without remainders;* and

d) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Students need exposure to various types of practical problems in which they must interpret the quotient and remainder based on the context. The chart below includes one example of each type of problem.	

MAKING SENSE OF THE REMAINDER IN DIVISION	
TYPE OF PROBLEM	EXAMPLE
Remainder is not needed and can be left over (or discarded).	Bill has 29 pencils to share fairly with 6 friends. How many pencils will each friend receive? 4 pencils with 5 pencils left over
Remainder is partitioned and represented as a fraction or decimal.	Six friends will share 29 ounces of juice. How many ounces will each person get if all of the juice is shared equally? $4\frac{5}{6}$ ounces
Remainder forces the answer to be increased to the next whole number.	There are 29 people going to the party by car. How many cars will be needed if each car holds 6 people? 5 cars
Remainder forces the answer to be rounded (giving an approximate answer).	Six children will share a bag of candy containing 29 pieces. About how many pieces of candy will each child get? about 5 pieces of candy

• Students will solve problems involving the division of decimals in grades five and six.

4.5 The student will
a) determine common multiples and factors, including least common multiple and greatest common factor;
b) add and subtract fractions and mixed numbers having like and unlike denominators;* and
c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A factor of a whole number is a whole number that divides evenly into that number with no remainder. A factor of a number is a divisor of the number. • A common factor of two or more numbers is a divisor that all of the numbers share. • The greatest common factor of two or more numbers is the largest of the common factors that all of the numbers share. • The product of the number and any natural number is a multiple of the number. • Common multiples and common factors can be useful when simplifying fractions. • The least common multiple of two or more numbers is the lowest number that is a multiple of all of the given numbers. • Estimation keeps the focus on the meaning of the numbers and operations, encourages reflective thinking, and helps build informal number sense with fractions. Students can reason with benchmarks to get an estimate without using an algorithm. • Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as 0, $\frac{1}{2}$, and 1. For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1. • Students should investigate addition and subtraction with fractions, using a variety of models (e.g., fraction circles, fraction strips, lines, pattern blocks). • While this standard requires instruction in solving problems with denominators of 2, 3, 4, 5, 6, 8, 10, and 12, students would benefit from experiences with other denominators. • When students use the least common multiple to determine common denominators to add or subtract fractions with unlike denominators, the least common multiple may be greater than 12, but will not exceed 60.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine common multiples and common factors of numbers. (a) • Determine the least common multiple and greatest common factor of no more than three numbers. (a) • Determine a common denominator for fractions, using common multiples. Common denominators should not exceed 60. (b) • Estimate the sum or difference of two fractions. (b, c) • Add and subtract fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. (Subtraction with fractions will be limited to problems that do not require regrouping). (b) • Solve single-step practical problems that involve addition and subtraction with fractions (proper or improper) and/or mixed numbers, having like and unlike denominators limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fraction. (Subtraction with fractions will be limited to problems that do not require regrouping). (c)

4.5 The student will
a) determine common multiples and factors, including least common multiple and greatest common factor;
b) add and subtract fractions and mixed numbers having like and unlike denominators;* and
c) solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). • Instruction involving addition and subtraction of fractions should include experiences with proper fractions, improper fractions, and mixed numbers as addends, minuends, subtrahends, sums, and differences. • A fraction is in simplest form when its numerator and denominator have no common factors other than one. The numerator can be greater than the denominator. • The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • At this level, denominators of fractions resulting from simplification will be limited to 12 or less.	

4.6 The student will
a) add and subtract decimals;* and
b) solve single-step and multistep practical problems involving addition and subtraction with decimals.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Addition and subtraction of decimals may be explored, using a variety of models (e.g., 10-by-10 grids, number lines, money). • The problem-solving process is enhanced when students create and solve their own practical problems and model problems using manipulatives and drawings. • In problem solving, emphasis should be placed on thinking and reasoning rather than on key words. Focusing on key words such as *in all, altogether, difference,* etc. encourages students to perform a particular operation rather than make sense of the context of the problem. It prepares students to solve a very limited set of problems and often leads to incorrect solutions. • The least number of steps necessary to solve a single-step problem is one.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate sums and differences of decimals. (a) • Add and subtract decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil. (a) • Solve single-step and multistep practical problems that involve adding and subtracting with decimals through thousandths. (b)

Students in grades three through five should be actively involved in measurement activities that require a dynamic interaction between students and their environment. Students can see the usefulness of measurement if classroom experiences focus on measuring objects and estimating measurements. Textbook experiences cannot substitute for activities that utilize measurement to answer questions about real problems.

The approximate nature of measurement deserves repeated attention at this level. It is important to begin to establish some benchmarks by which to estimate or judge the size of objects.

Students use standard and nonstandard, age-appropriate tools to measure objects. Students also use age-appropriate language of mathematics to verbalize the measurements of length, weight/mass, liquid volume, area, perimeter, temperature, and time.

The focus of instruction should be an active exploration of the real world in order to apply concepts from the two systems of measurement (metric and U.S. Customary), to measure length, weight/mass, liquid volume/capacity, area, perimeter, temperature, and time. Students' understanding of measurement continues to be enhanced through experiences using appropriate tools such as rulers, balances, clocks, and thermometers.

The study of geometry helps students represent and make sense of the world. In grades three through five, reasoning skills typically grow rapidly, and these skills enable students to investigate geometric problems of increasing complexity and to study how geometric terms relate to geometric properties. Students develop knowledge about how geometric figures relate to each other and begin to use mathematical reasoning to analyze and justify properties and relationships among figures.

Students discover these relationships by constructing, drawing, measuring, comparing, and classifying geometric figures. Investigations should include explorations with everyday objects and other physical materials. Exercises that ask students to visualize, draw, and compare figures will help them not only to develop an understanding of the relationships, but to develop their spatial sense as well. In the process, definitions become meaningful, relationships among figures are understood, and students are prepared to use these ideas to develop informal arguments.

Students investigate, identify, draw representations of, and describe the relationships among points, lines, line segments, rays, and angles. Students apply generalizations about lines, angles, and triangles to develop understanding about congruence; parallel, intersecting, and perpendicular lines; and classification of triangles.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)
- **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

4.7 The student will solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.

Understanding the Standard	Essential Knowledge and Skills
• Perimeter is the path or distance around any plane figure. • To determine the perimeter of any polygon, determine the sum of the lengths of the sides. • Area is the surface included within a plane figure. Area is measured by the number of square units needed to cover a surface or plane figure. • Students should have opportunities to investigate and discover, using manipulatives, the formulas for the area of a square and the area of a rectangle. - Area of a square = side length × side length - Area of rectangle = length × width • Perimeter and area should always be labeled with the appropriate unit of measure.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the perimeter of a polygon with no more than eight sides, when the lengths of the sides are given, with diagrams. • Determine the perimeter and area of a rectangle when given the measure of two adjacent sides, with and without diagrams. • Determine the perimeter and area of a square when the measure of one side is given, with and without diagrams. • Solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.

4.8 The student will
a) estimate and measure length and describe the result in U.S. Customary and metric units;
b) estimate and measure weight/mass and describe the result in U.S. Customary and metric units;
c) given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and
d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

Understanding the Standard	Essential Knowledge and Skills
• The measurement of an object must include the unit of measure along with the number of iterations. • Length is the distance between two points along a line. • U.S. Customary units for measurement of length include inches, feet, yards, and miles. Appropriate measuring devices include rulers, yardsticks, and tape measures. • Metric units for measurement of length include millimeters, centimeters, meters, and kilometers. Appropriate measuring devices include centimeter rulers, meter sticks, and tape measures. • When measuring with U.S. Customary units, students should be able to measure to the nearest part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), foot, or yard. • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • Balances are appropriate measuring devices to measure weight in U.S. Customary units (ounces, pounds) and mass in metric units (grams, kilograms). • Practical experience measuring the weight/mass of familiar objects (e.g., foods, pencils, book bags, shoes) helps to establish benchmarks and facilitates the student's ability to estimate weight/mass. • Students should measure the liquid volume of everyday objects in U.S. Customary units, including cups, pints, quarts, gallons, and record the volume including the appropriate unit of measure (e.g., 24 gallons).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an appropriate unit of measure (inch, foot, yard, mile, millimeter, centimeter, and meter) to use when measuring length in both U.S. Customary and metric units. (a) • Estimate and measure length in U.S. Customary and metric units, measuring to the nearest part of an inch ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$), and to the nearest foot, yard, millimeter, centimeter, or meter, and record the length including the unit of measure (e.g., 24 inches). (a) • Compare estimates of the length with the actual measurement of the length. (a) • Determine an appropriate unit of measure (ounce, pound, gram, and kilogram) to use when measuring the weight/mass of everyday objects in both U.S. Customary and metric units. (b) • Estimate and measure the weight/mass of objects in both U.S. Customary and metric units (ounce, pound, gram, or kilogram) to the nearest appropriate measure, using a variety of measuring instruments. (b) • Record the weight/mass of an object with the unit of measure (e.g., 24 grams). (b)

4.8 The student will

a) estimate and measure length and describe the result in U.S. Customary and metric units;

b) estimate and measure weight/mass and describe the result in U.S. Customary and metric units;

c) given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and

d) solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

Understanding the Standard	Essential Knowledge and Skills
• Students at this level will be given the equivalent measure of one unit when asked to determine equivalencies between units in the U.S. Customary system. – For example, students will be told one gallon is equivalent to four quarts and then will be asked to apply that relationship to determine: - the number of quarts in five gallons; - the number of gallons equal to 20 quarts; - When empty, Tim's 10-gallon container can hold how many quarts?; or - Maria has 20 quarts of lemonade. How many empty one-gallon containers will she be able to fill?	• Given the equivalent measure of one unit, identify equivalent measures between units within the U.S. Customary system for: – length (inches and feet, feet and yards, inches and yards); yards and miles; – weight/mass (ounces and pounds); and – liquid volume (cups, pints, quarts, and gallons). (c) • Solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units. (d)

4.9 The student will solve practical problems related to elapsed time in hours and minutes within a 12-hour period.

Understanding the Standard	Essential Knowledge and Skills
• Elapsed time is the amount of time that has passed between two given times. • Elapsed time should be modeled and demonstrated using analog clocks and timelines. • Elapsed time can be found by counting on from the beginning time or counting back from the ending time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve practical problems related to elapsed time in hours and minutes, within a 12-hour period (within a.m., within p.m., and across a.m. and p.m.): – when given the beginning time and the ending time, determine the time that has elapsed; – when given the beginning time and amount of elapsed time in hours and minutes, determine the ending time; or – when given the ending time and the elapsed time in hours and minutes, determine the beginning time.

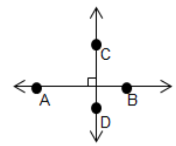
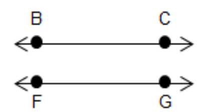
4.10 The student will

a) identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and

b) identify and describe intersecting, parallel, and perpendicular lines.

Understanding the Standard	Essential Knowledge and Skills
• Points, lines, line segments, rays, and angles, including endpoints and vertices are fundamental components of noncircular geometric figures. • A point is a location in space. It has no length, width, or height. A point is usually named with a capital letter. • The shortest distance between two points in a plane, a flat surface, is a line segment. • A line is a collection of points extending infinitely in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. Arrows must be drawn to show that the line goes on infinitely in both directions (e.g., $\overleftrightarrow{AB}$ read as "line AB"). • A line segment is part of a line. It has two endpoints and includes all the points between and including the endpoints. To name a line segment, name the endpoints (e.g., $\overline{AB}$ read as "line segment AB"). • A ray is part of a line. It has one endpoint and extends infinitely in one direction. To name a ray, say the name of its endpoint first and then say the name of one other point on the ray (e.g., $\overrightarrow{AB}$ read as "ray AB"). • An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect. • An angle can be named in three different ways by using: – three letters in order: a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • A vertex is the point at which two lines, line segments, or rays meet to form an angle. In solid figures, a vertex is the point at which three or more edges meet. • Lines in a plane either intersect or are parallel. Perpendicularity is a special case of intersection. • Intersecting lines have one point in common.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices. (a) • Use symbolic notation to name points, lines, line segments, rays, and angles. (a) • Identify parallel, perpendicular, and intersecting line segments in plane and solid figures. (b) • Identify practical situations that illustrate parallel, intersecting, and perpendicular lines. (b) • Use symbolic notation to describe parallel lines and perpendicular lines. (b)

4.10 The student will
a) identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and
b) identify and describe intersecting, parallel, and perpendicular lines.

Understanding the Standard	Essential Knowledge and Skills
• Perpendicular lines intersect at right angles. The symbol ⊥ is used to indicate that two lines are perpendicular. For example, the notation $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ is read as "line AB is perpendicular to line CD."  $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ • Students need experiences using geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. • Parallel lines lie in the same plane and never intersect. Parallel lines are always the same distance apart and do not share any points. The symbol ∥ indicates that two or more lines are parallel. For example, the notation $\overleftrightarrow{BC} \parallel \overleftrightarrow{FG}$ is read as "line BC is parallel to line FG".  $\overleftrightarrow{BC} \parallel \overleftrightarrow{FG}$	

4.11 The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces) using concrete models and pictorial representations.

Understanding the Standard

- The study of geometric figures must be active, using visual images and concrete materials (tools such as graph paper, pattern blocks, geoboards, geometric solids, and computer software tools).
- Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures.
- A plane figure is any closed, two-dimensional shape.
- A solid figure is three-dimensional, having length, width, and height.
- A face is any flat surface of a solid figure.
- An angle is formed by two rays with a common endpoint called the *vertex*. Angles are found wherever lines and/or line segments intersect.
- An edge is the line segment where two faces of a solid figure intersect.
- A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. In solid figures, a vertex is the point at which three or more faces meet.
- A cube is a solid figure with six congruent, square faces. All edges are the same length. A cube has eight vertices and 12 edges.
- A rectangular prism is a solid figure in which all six faces are rectangles. A rectangular prism has eight vertices and 12 edges. A cube is a special case of a rectangular prism.
- A sphere is a solid figure with all of its points the same distance from its center.
- A square pyramid is a solid figure with a square base and four faces that are triangles with a common vertex. A square pyramid has five vertices and eight edges.
- Characteristics of solid figures included at this grade level are defined in the chart below:

Solid Figure	# of Faces	Shape of Faces	# of Edges	# of Vertices
Cube	6	Squares	12	8
Rectangular Prism	6	Rectangles	12	8
Square Pyramid	5	Square/Triangles	8	5
Sphere	0	N/A	0	0

Essential Knowledge and Skills

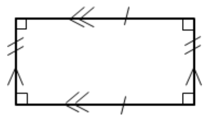
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify concrete models and pictorial representations of solid figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder).
- Identify and describe solid figures (cube, rectangular prism, square pyramid, and sphere) according to their characteristics (number of angles, vertices, edges, and by the number and shape of faces).
- Compare and contrast plane and solid figures (circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular prism) according to their characteristics (number of sides, angles, vertices, edges, and the number and shape of faces).

4.12 The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
<tr><td>

- A quadrilateral is a polygon with four sides.
- A parallelogram is a quadrilateral with both pairs of opposite sides parallel and congruent.
- Congruent figures have the same size and shape. Congruent sides are the same length.
- A rectangle is a quadrilateral with four right angles, and, opposite sides that are parallel and congruent.
- The geometric markings shown on the rectangle below indicate parallel sides with an equal number of arrows and congruent sides indicated with an equal number of hatch (hash) marks.



- A square is a rectangle with four congruent sides and four right angles.
- A trapezoid is a quadrilateral with exactly one pair of parallel sides.
- A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following:
  - opposite sides are congruent
  - opposite sides are parallel
  - opposite angles are congruent

</td><td>

The student will use problem solving, mathematical communication, mathematical reasoning, connections and representation to

- Develop definitions for parallelograms, rectangles, squares, rhombi, and trapezoids.
- Identify properties of quadrilaterals including parallel, perpendicular, and congruent sides.
- Classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.
- Compare and contrast the properties of quadrilaterals.
- Identify parallel sides, congruent sides, and right angles using geometric markings to denote properties of quadrilaterals.

</td></tr>
</table>

Students entering grades three through five have begun to explore the concept of the measurement of chance and are able to determine possible outcomes of given events. Students have utilized a variety of random generator tools, including random number generators (number cubes), spinners, and two-sided counters. In game situations, students have had initial experiences in predicting whether a game is fair or not fair. Furthermore, students are able to identify events as likely or unlikely to happen. Thus the focus of instruction in grades three through five is to deepen their understanding of the concepts of probability by

- offering opportunities to set up models simulating practical events;
- engaging students in activities to enhance their understanding of fairness; and
- engaging students in activities that instill a spirit of investigation and exploration and providing students with opportunities to use manipulatives.

The focus of statistics instruction is to assist students with further development and investigation of data collection strategies. Students should continue to focus on:

- posing questions;
- collecting data and organizing this data into meaningful graphs, charts, and diagrams based on issues relating to practical experiences;
- interpreting the data presented by these graphs;
- answering descriptive questions (“How many?” “How much?”) from the data displays;
- identifying and justifying comparisons (“Which is the most? Which is the least?” “Which is the same? Which is different?”) about the information;
- comparing their initial predictions to the actual results; and
- communicating to others their interpretation of the data.

Through a study of probability and statistics, students develop a real appreciation of data analysis methods as powerful means for decision making.

4.13 The student will

a) determine the likelihood of an outcome of a simple event;

b) represent probability as a number between 0 and 1, inclusive; and

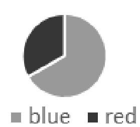
c) create a model or practical problem to represent a given probability.

Understanding the Standard

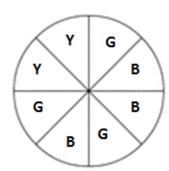
- A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives.
- Probability is the measure of likelihood that an event will occur. An event is a collection of outcomes from an investigation or experiment.
- The terms *certain*, *likely*, *equally likely, unlikely*, and *impossible* can be used to describe the likelihood of an event. If all outcomes of an event are equally likely, the probability of an event can be expressed as a fraction, where the numerator represents the number of favorable outcomes and the denominator represents the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event is equal to:

$$\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes.}}$$

- Probability is quantified as a number between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., if eight balls are in a bag, four yellow and four blue, there is zero probability that a red ball could be selected). An event is "certain" if it has a probability of one (e.g., the probability that if 10 coins, all pennies, are in a bag that it is certain a penny could be selected).
- For an event such as flipping a coin, the things that can happen are called *outcomes*. For example, there are two possible outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. The two possible outcomes, heads up or tails up, are equally likely.
- For another event such as spinning a spinner that is one-third red and two-thirds blue, the two outcomes, red and blue, are not equally likely.



- Equally likely events can be represented with fractions of equivalent value. For example, on a spinner with eight sections of equal size, where three of the sections are labeled G (green) and three are labeled B (blue), the chances of landing on green or on blue are equally likely; the probability of each of these events is the same, or $\frac{3}{8}$.

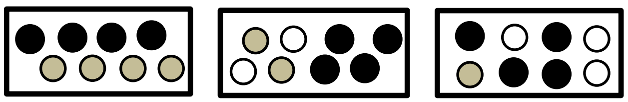


Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Model and determine all possible outcomes of a given simple event where there are no more than 24 possible outcomes, using a variety of manipulatives (e.g., coins, number cubes, and spinners). (a)
- Determine the outcome of an event that is least likely to occur or most likely to occur where there are no more than 24 possible outcomes. (a)
- Write the probability of a given simple event as a fraction, where there are no more than 24 possible outcomes. (b)
- Determine the likelihood of an event occurring and relate it to its whole number or fractional representation (e.g., impossible or zero; equally likely; certain or one). (a, b)
- Create a model or practical problem to represent a given probability. (c)

4.13 The student will
a) determine the likelihood of an outcome of a simple event;
b) represent probability as a number between 0 and 1, inclusive; and
c) create a model or practical problem to represent a given probability.

Understanding the Standard	Essential Knowledge and Skills
• Students need opportunities to create a model or practical problem that represents a given probability. For example, if asked to create a box of marbles where the probability of selecting a black marble is $\frac{4}{8}$, sample responses might include:  • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time).	

4.14 The student will
a) collect, organize, and represent data in bar graphs and line graphs;
b) interpret data represented in bar graphs and line graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

Understanding the Standard	Essential Knowledge and Skills
• Data analysis helps describe data, recognize patterns or trends, and make predictions. • Investigations involving practical data should occur frequently; data can be collected through brief class surveys or through more extended projects taking many days. • Students formulate questions, predict answers to questions under investigation, collect and represent initial data, and consider whether the data answer the questions. • There are two types of data: categorical (e.g., qualitative) and numerical (e.g., quantitative). Categorical data are observations about characteristics that can be sorted into groups or categories, while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data. • Bar graphs display grouped data such as categories using rectangular bars whose length represents the quantity the bar represents. Bar graphs should be used to compare counts of different categories (categorical or qualitative data). Grid paper can assist students in creating graphs with greater accuracy. – A bar graph uses horizontal or vertical bars to represent counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis that displays the scale representing the count for the categories should begin at zero and extend one increment above the greatest recorded piece of data. Grade four students should collect and represent data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, 10, or 100. – Each axis should be labeled, and the graph should be given a title. • Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences, least and greatest, the categories, and total number of responses) should be written.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect data, using, for example, observations, measurement, surveys, scientific experiments, polls, or questionnaires. (a) • Organize data into a chart or table. (a) • Represent data in bar graphs, labeling one axis with equal whole number increments of one or more (numerical data) (e.g., 2, 5, 10, or 100) and the other axis with categories related to the title of the graph (categorical data) (e.g., swimming, fishing, boating, and water skiing as the categories of “Favorite Summer Sports”). (a) • Represent data in line graphs, labeling the vertical axis with equal whole number increments of one or more and the horizontal axis with continuous data commonly related to time (e.g., hours, days, months, years. Line graphs will have no more than 10 identified points along a continuum for continuous data. (a) • Title the graph or identify an appropriate title. Label the axes or identify the appropriate labels. (a) • Interpret data by making observations from bar graphs and line graphs by describing the characteristics of the data and the data as a whole (e.g., the time period when the temperature increased the most, the category with the greatest/least, categories with the same number of responses, similarities and differences, the total number). One set of data will be represented on a graph. (b)

4.14 The student will
a) collect, organize, and represent data in bar graphs and line graphs;
b) interpret data represented in bar graphs and line graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

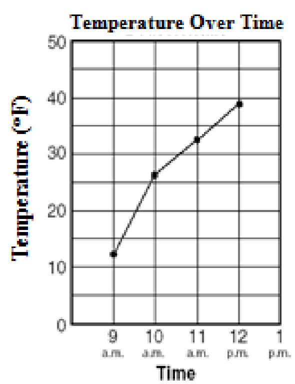
Understanding the Standard	Essential Knowledge and Skills
• Line graphs are used to show how two data sets (numerical or quantitative data) are related. Line graphs may be used to show how one variable changes over time (numerical or quantitative data). By looking at a line graph, it can be determined whether the change in the data set is increasing, decreasing, or staying the same over time. – The values along the horizontal axis represent continuous data, usually some measure of time (e.g., time in years, months, or days). The data presented on a line graph is referred to as "continuous data," as it represents data collected over a continuous period of time. – The values along the vertical axis represent the range of values in the collected data set at the given time interval on the horizontal axis. The scale values on the vertical axis should represent equal increments of multiples of whole numbers, fractions, or decimals, depending upon the data being collected. The scale should extend one increment above the greatest recorded piece of data. – Plot a point to represent the data collected for each time increment. Use line segments to connect the points in order moving left to right. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph should be included (e.g., trends of increase and/or decrease, and least and greatest). • For example, a line graph documenting data gathered during a planting cycle might show length of time and the height of a plant at any given interval. • Different situations call for different types of graphs. The way data are displayed is often dependent upon what someone is trying to communicate. • Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different aspects of the same data. Following construction of graphs, students benefit from discussions around what information each graph provides. • Tables or charts organize the exact data and display numerical information. They do not show visual comparisons, which generally means it takes longer to understand or to examine trends.	• Interpret data by making inferences from bar graphs and line graphs. (b) • Interpret the data to answer the question posed, and compare the answer to the prediction (e.g., "The summer sport preferred by most is swimming, which is what I predicted before collecting the data."). (b) • Write at least one sentence to describe the analysis and interpretation of the data, identifying parts of the data that have special characteristics, including categories with the greatest, the least, or the same. (b) • Compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph; a chart and a line graph; a pictograph and a bar graph). (c)

4.14 The student will
a) collect, organize, and represent data in bar graphs and line graphs;
b) interpret data represented in bar graphs and line graphs; and
c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

Understanding the Standard	Essential Knowledge and Skills
• Line graphs display data that changes continuously over time. This allows overall increases or decreases to be seen more readily. • Bar graphs can be used to compare data easily and see relationships. They provide a visual display comparing the numerical values of different categories. The scale of a bar graph may affect how one perceives the data. • Examples of some questions that could be explored in comparing a chart to a line graph include: In which representation do you readily see the increase or decrease of temperature over time? In which representation is it easiest to determine when the greatest rise in temperature occurred?	

Temperature Over Time

Time	Temperature
9 a.m.	12
10 a.m.	26
11 a.m.	33
12 p.m.	39

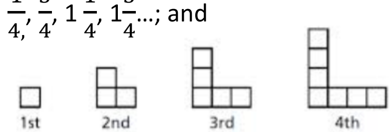


Students entering grades three through five have had opportunities to identify patterns within the context of the school curriculum and in their daily lives, and they can make predictions about them. They have had opportunities to use informal language to describe the changes within a pattern and to compare two patterns. Students have also begun to work with the concept of a variable by describing mathematical relationships within a pattern.

The focus of instruction is to help students develop a solid use of patterning as a problem solving tool. At this level, patterns are represented and modeled in a variety of ways, including numeric, geometric, and algebraic formats. Students develop strategies for organizing information more easily to understand various types of patterns and functional relationships. They interpret the structure of patterns by exploring and describing patterns that involve change, and they begin to generalize these patterns. By interpreting mathematical situations and models, students begin to represent these, using symbols and variables to write “rules” for patterns, to describe relationships and algebraic properties, and to represent unknown quantities.

4.15 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.

Understanding the Standard

- Patterns and functions can be represented in many ways and described using words, tables, graphs, and symbols.
- Patterning activities should involve making connections between concrete materials and numerical representations (e.g., number sequence, table, description). Numeric patterns, at this level, will include both growing and repeating patterns (limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less).
- Students need experiences with growing patterns using concrete materials and calculators.
- Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing the relationship symbolically or algebraically.
- Sample growing patterns that are, or can be, represented as numerical (arithmetic) growing patterns include:
  - 2, 4, 8, 16, …;
  - 8, 10, 13, 17, …;
  - $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}$…; and
  - 

- Students in grade three had experiences working with input/output tables. At this level, input/output tables should be analyzed for a pattern to determine an unknown value or describe the rule that explains how to find the output when given the input. Determining and applying rules builds the foundation for functional thinking. Sample input/output tables that require determination of the rule or missing terms can be found below:

Rule: ?	
Input	Output
4	11
5	12
6	13
10	17

Rule: ?	
Input	Output
145	130
100	85
75	60
50	?

Rule: ?	
Input	Output
2	8
4	16
?	20
8	32

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Identify and describe patterns, using words, objects, pictures, numbers, and tables.
- Create patterns using objects, pictures, numbers, and tables.
- Extend patterns, using objects, pictures, numbers, and tables.
- Solve practical problems that involve identifying, describing, and extending single-operation input and output rules, limited to addition, subtraction, and multiplication of whole numbers and addition and subtraction of fractions with like denominators of 12 or less.
- Identify the rule in a single-operation numerical pattern found in a list or table, limited to addition, subtraction, and multiplication of whole numbers.

4.16 The student will recognize and demonstrate the meaning of equality in an equation.

Understanding the Standard	Essential Knowledge and Skills
• Mathematical relationships can be expressed using equations. • An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal symbol (e.g., 8, 15 × 12). • An equation represents the relationship between two expressions of equal value (e.g., 12 × 3 = 72 ÷ 2). • The equal-symbol (=) means that the values on either side are equivalent (balanced). • The not equal symbol (≠) means that the values on either side are not equivalent (not balanced).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation to represent the relationship between equivalent mathematical expressions (e.g., 4 × 3 = 2 × 6; 10 + 8 = 36 ÷ 2; 12 × 4 = 60 – 12). • Identify and use the appropriate symbol to distinguish between expressions that are equal and expressions that are not equal, using addition, subtraction, multiplication, and division (e.g., 4 × 12 = 8 × 6 and 64 ÷ 8 ≠ 8 × 8).